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COMPRESSION OF A PLANE WITH A CRACK AND AN INCLUSION

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The paper deals with the problem of compression of a plane containing a crack and a circular inclusion. The inclusion and the plane have different Poisson's ratios, but the same shear modulus. The homogeneous compressive load at infinity is parallel to the direction of the cut. Analytic solutions of the elastic problems for the cases of a free and a partly compressed crack are given, the stress intensity at the tips are found and the character of the crack propagation in such a model studied. The results obtained agree with the experimental data obtained during fracture of concrete.

Compressive fracture was studied from the point of view of brittle fracture by Cherepanov. The pinch effect was analyzed in [1], while in [2] the author applied to the phenomenon of a mine shock the problem of a crack in a homogeneous elastic plane under the action of a compressive load, with an overlap of the edges and an arbitrary law of friction operating between these edges.

In composite materials which can be represented in the form of a matrix with randomly distributed inclusions, the cracks propagate in the direction parallel to the line of action of the compressive load. It is therefore natural to consider the following problem. An elastic plane with a cut and an elastic circular inclusion is compressed at infinity along the direction of the cut. Without an inclusion, the crack will not grow. In the presence of an inclusion, stress concentrations will appear at the crack tips and the crack may begin to grow.

It should be noted that the influence of other type defects, namely the growth of cracks originating at the pores in a matrix, was studied e.g. in [3,4].

A similar model was investigated earlier in connection with tensile fracture of fibrous composites (see e.g. [5,6]). The most general consideration of the problem is given in [7] where a numerically solvable singular integral equation is given for the derivative of a function equal to the value of the opening between the crack edges.

1. Solution of the elastic problem. Let us consider an elastic plane OXY containing a cut along the segment $[a, b]$ of the OX- axis and a circular inclusion S

$$S = \{z, \mid z - ih \mid \leqslant R\}, \quad z = x + iy, \quad h \geqslant 0, R > 0$$

We shall assume that the cut edges are stress free, that continuous forces and displacements are present at the boundary L of the inclusion and, that a homogeneous compressive stress acts at infinity in the direction of the OX- axis. This leads to the following boundary value problem:

$$z \in L: [p_{in}] = [u_i] = 0 \tag{1.1}$$

$$z \in [a, b]: p_{12} = p_{22} = 0 \tag{1.2}$$

$$z \to \infty: p_{11} \to p < 0, \quad p_{12} \to 0, \; p_{22} \to 0$$

Here p_{ij} and u_i are the components of the stress tensor and displacement vector, respectively, and [f] denotes the jump in the value of the function during the passage across the corresponding line.

Let us introduce the Kolosov–Muskhelishvili potentials: φ_i and ψ_i for S, φ_0 and ψ_0 for the matrix, $\varphi^\circ = pz/4$, $\psi^\circ = -pz/2$ for the stressed state at infinity. We set $\varphi_0 = \varphi^\circ + \varphi_1$, $\psi_0 = \psi^\circ + \psi_1$. Following the idea of Sherman [8], we express the potentials in terms of functions holomorphic outside the cut

$$\varphi = \begin{cases} \varphi_1, & z \overline{\in} S \\ \dfrac{\varkappa_i + 1}{\varkappa + 1}\varphi_i - \varphi^\circ + C + D, & z \in S \end{cases} \tag{1.3}$$

$$\psi = \begin{cases} \psi_1 - \varepsilon G - \varepsilon G^\circ, & z \overline{\in} S \\ \psi_i - \varepsilon G - \varepsilon G^\circ + \dfrac{\varkappa_i \overline{C} - \overline{D}}{\varkappa + 1}, & z \in S \end{cases}$$

$$G(z) = \frac{1}{2\pi i}\int_L \frac{\overline{\varphi} + \overline{s}\varphi'}{s - z}\, ds$$

$$G^0(z) = \frac{1}{2\pi i}\int_L \frac{\overline{\varphi^\circ} + \overline{s}\varphi^{\circ\prime}}{s - z}\, ds = \begin{cases} \dfrac{1}{2} iph, & z \in S \\ \dfrac{\frac{1}{2} pR^2}{z - ih}, & z \overline{\in} S \end{cases}$$

$$\varepsilon = \frac{\varkappa - \varkappa_i}{\varkappa_i + 1} = \begin{cases} \dfrac{\nu_i - \nu}{1 - \nu_i} & \text{(plane deformation)} \\ \dfrac{\nu_i - \nu}{1 + \nu} & \text{(plane stressed state)} \end{cases}$$

Here ν_i and ν are the Poisson's ratios of the inclusion and the matrix, respectively, C and D are constants determined from the condition of uniqueness of the displacements, and $\varphi(\infty) = \psi(\infty) = 0$.

Using (1.2), we obtain the boundary value problem for the outside of the cut $[a, b]$

$$\Phi + \overline{\Phi} + t\overline{\Phi}' + \overline{\Psi} + \varepsilon\overline{G}' + \varepsilon\overline{G}^{\circ\prime} = 0 \tag{1.4}$$
$$\Phi = \varphi', \quad \Psi = \psi', \quad \Phi(\infty) = O(z^{-2}), \quad \Psi(\infty) = O(z^{-2})$$

Outside the circle S, we have

$$G(z) = \overline{\varphi}\left(-ih + \frac{R^2}{z - ih}\right) - \overline{\varphi(ih)} + \frac{R^2\,\Phi(ih)}{z - ih} \tag{1.5}$$

For the usual range of values of the Poisson's ratio $1/4 \leqslant \nu \leqslant 1/2$, we have: in the case of plane deformation $1/3 \leqslant \varepsilon \leqslant 1/2$, and in the case of a plane state of stress we have $1/6 \leqslant \varepsilon \leqslant 1/5$ A set of the Poisson's ratios characteristic of e.g. concrete, yields the values of ε ranging from 0.08 to 0.15. For this reason we shall assume ε to be small and seek a solution of the problem (1.4) in the form of a series in powers of ε

$$\Phi(z, \varepsilon) = \sum_{n=1}^{\infty} \Phi_n(z)\varepsilon^n, \quad \Psi(z, \varepsilon) = \sum_{n=1}^{\infty} \Psi_n(z)\varepsilon^n$$

Then for each term of that series we have the well known boundary value problem

$$\Phi_n + \overline{\Phi}_n + t\overline{\Phi_n'} + \overline{\Psi}_n = -\overline{G_{n-1}'}$$

The function G_0 is known for $n = 1$, and for $n > 1$ we can find G_{n-1} by substituting Φ_{n-1} into (1.5).

We have the following recurrence relation for Φ_n :

$$\Phi_n(z) = -\frac{1}{2\pi i X(z)} \int_a^b \frac{X(t)\, G_{n-1}'(t)}{t - z}\, dt$$

$$X(z) = \sqrt{(z-a)(z-b)}, \quad \lim_{z\to\infty} \frac{X(z)}{z} = 1$$

Any number of terms of this series can be obtained in finite form. This condition of convergence in the Hölder space with index $1/2$ is easily obtained (see [9], Sect. 133). The condition has the form $\varepsilon A < 1$ where A is a constant depending on the geometrical parameters of the problem

$$A = \left(\frac{1}{2} + C_{0,5}\right) \frac{\rho \sqrt[3]{\delta}}{1-\rho} \left(\frac{1}{2} + 2\sqrt{\delta}\right) \left[1 + 2\sqrt{\delta\rho^2\left(1 + \frac{b^2}{h^2}\right)}\right] \times$$
$$\left\{\frac{1-\rho}{\sqrt{1 + a^2/h^2}} + \left[1 + \frac{\rho^2\sqrt{\delta}}{(1-\rho)^2}\right]^2\right\}, \quad \rho = \frac{R}{h}, \quad \delta = \frac{l}{R}$$

(l denotes the length of the cut)

When $h < R$, then h should be replaced by $\sqrt{h^2 + a^2}$.

It should be noted that for short length cuts and for cuts away from the inclusion, A is small, i.e. in these cases the condition of convergence still holds when ε are not small; at small ε the convergence becomes rapid. Therefore, in what follows, we shall limit ourselves to the first approximation

$$\Phi = \frac{\varepsilon p R^2}{4X(z)} \left\{\frac{X(z)}{(z + ih)^2} + \frac{(2ih + a + b)(z + ih) - 2(ih + a)(ih + b)}{2X(-ih)(z + ih)^2}\right\}$$

Now we can write all the remaining functions. We shall only give the formulas for the stress concentration factors which will be needed later

$$(a + ih) K^a = (b + ih) K^b = -\frac{\varepsilon p R^2 \sqrt{2\pi (b-a)}}{4\sqrt{(ih + a)(ih + b)}} \tag{1.7}$$

$$K^{a,b} = K_{\mathrm{I}}^{a,b} - iK_{\mathrm{II}}^{a,b}$$

The approximate solution obtained has a simple physical meaning. The cut in the infinite homogeneous plane is acted upon by a stress field which arises in a plane with an inclusion but without a cut, with the potentials

$$\Phi = 0, \quad \Psi = -\varepsilon G^{\circ\prime} = \frac{\varepsilon p R^2}{2(z - ih)^2} \tag{1.8}$$

Obviously, such an approximation holds for a cut sufficiently distant from the inclusion, for any combination of the elastic models and for any shape of the inclusion. The potentials of the corresponding state of stress can be written in the form

$$\Phi = \frac{\Phi_2}{(z - ih)^2} + O(z^{-3}), \quad \Psi = \frac{\Psi_2}{(z - ih)^2} + O(z^{-3})$$

The term proportional to Φ_2 produces in the boundary condition a term of order $O(z^{-3})$, and the term proportional to Ψ_2 produces a term of order $O(z^{-2})$. Therefore, in the first approximation it is sufficient to consider the potentials of the form (1.8). By inspecting the dimensions we find that $\Psi_2 = kpR^2$ where R is the characteristic size of the inclusion and k is a coefficient depending on the form and on the elastic moduli. Such representations was used in [10] to account for the influence of a unit cut, in the investigation of a plane with a doubly periodic system of cuts.

2. Analysis of the solution. The cut edges must not overlap if the solution (1.6) is to have physical meaning. From the equations $K_I^a = K_I^b = 0$, we obtain

$$a = b \tag{2.1}$$

$$b = \frac{a(a^2 - 3h^2)}{h^2 - 3a^2}, \quad |a| > \frac{h}{\sqrt{3}} \tag{2.2}$$

$$a = \frac{b(b^2 - 3h^2)}{h^2 - 3b^2}, \quad |b| > \frac{h}{\sqrt{3}} \tag{2.3}$$

Let us set, for definiteness, $\varepsilon > 0$, i.e. $\nu_i > \nu$ (such a relation holds for heavy concretes). Then the conditions $K_I^a \geqslant 0$ and $K_I^b \geqslant 0$ yields the following two admissible regions for the cut tips:

$$0 \leqslant \frac{h}{\sqrt{3}} \leqslant a_1(b) \leqslant a \leqslant b, \qquad a \leqslant b \leqslant b_1(a) \leqslant -\frac{h}{\sqrt{3}} \leqslant 0$$

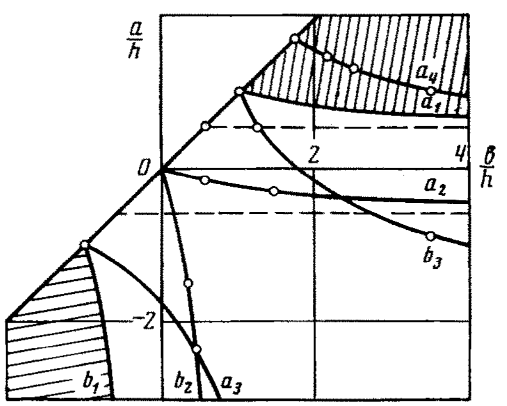

Fig.1

Figure 1 depicts these regions in the plane of variables b, a. The lines a_1, a_2 and a_3 are branches of the curve defined by (2.2), and b_1, b_2, b_3 are defined by (2.3). The admissible regions are shaded.

The local state of stress at the cut tips is a combination of tensile stress and transverse shear. When the cut moves away from the inclusion or approaches the OX - axis, the quantities K_{II}^a and K_{II}^b (i.e. the contributions of the shear to the local state of stress at the tips tend to zero. In the particular case of $h = 0$ (the cut lies at the extension of the diameter) we have

$$K^{a,b} = K_I^{a,b}, \quad 4\sqrt{a^3 b} K^a = 4\sqrt{ab^3} K^b = -\varepsilon p R^2 \sqrt{2\pi(b - a)}$$

In what follows we shall consider, without loss of generality, the cuts for which $b \geqslant |a| \geqslant 0$. We have for these cuts $|K^a| > |K^b|$. If we adopt the conditions of fracture in the form $|K^a| = K_c$, $|K^b| = K_c$ where K_c is the material constant of the matrix, then the condition of fracture will first be fulfilled at the tips $z = a$. The critical load is found from the formula

$$p = -2K_c \frac{[(a^2 + h^2)^3 (b^2 + h^2)]^{1/4}}{\varepsilon R^2 \sqrt{2\pi(b - a)}}$$

For small length cracks the numerator in the above expression can be replaced by a^2+h^2 The critical load has the following minimum value for all admissible cuts:

$$p_{\min} = -\frac{4K_c h\sqrt{h}}{\varepsilon R^2 \sqrt{3\pi\sqrt{3}}}$$

We note that under the formulation used here the crack changes its initial direction after the critical load has been reached. In a real material the cracks grow along the line of application of the load, and this can be explained by the presence of many inclusions. We can consider the following symmetric problem as the simplest model. We have two identical inclusions. The initial cut lies on a line passing through the middle of the segment connecting the centers of the inclusions, in the direction perpendicular to this segment. In this case it is clear that we have, within the same accuracy,

$$K_{(2)}^{a,b} = 2K_{\mathrm{I}}^{a,b}$$

where $K_{(2)}^{a,b}$ are the stress concentration coefficients at the cut tips in the symmetric problem. By symmetry, the cut will grow in its initial direction. The regions within which free cuts may exist, remain as before.

The equation $K_{\mathrm{I}}^a = K_{\mathrm{I}}^b$ has two solutions (at $b \geqslant |a| \geqslant 0$):

$$a = b, \quad a = a_4(b) = h(b + h\sqrt{3}) / (\sqrt{3}b - h)$$

For small cracks we have $K_{\mathrm{I}}^a > K_{\mathrm{I}}^b$, therefore in the region $a_4 < a < b$ we have $K_{\mathrm{I}}^a > 0$, while in the region $a_1 < a < a_4$, $K_{\mathrm{I}}^a < 0$. We have the following relations:

$$\frac{\partial K_{\mathrm{I}}^b}{\partial a} = -\frac{K_{\mathrm{I}}^a}{2(b-a)} < 0, \quad \frac{\partial K_{\mathrm{I}}^a}{\partial b} = \frac{K_{\mathrm{I}}^b}{2(b-a)} > 0$$

i.e. the quantity K_{I}^a increases with increasing b and attains its maximum value when the cut is semiinfinite

$$K_{\mathrm{I}}^a = -\frac{\varepsilon p R^2}{4}\left[\pi\,\frac{a^3 - 3ah^2 + (a^2+h^2)^{3/2}}{(a^2+h^2)^3}\right]^{1/2}$$

Similarly, the quantity K_{I}^b as a function of a , attains its maximum on the line $a_1(b)$

$$K_{\mathrm{I}}^b = +\tfrac{1}{2}\varepsilon p R^2 (h^2 - 3a^2)\,[2\pi a^3 (h^2 - a^2)^3 (a^2 + h^2)^{-9}]^{1/4}$$

Since $K_{\mathrm{I}}^a = 0$ when $a = b$ and $a = a_1(b)$, it follows that K_{I}^a, as a function of a , attains a maximum on at least one line lying between the above lines. Similarly, K_{I}^b as a function of b, decreases monotonously at sufficiently large b .

The properties of the stress concentration coefficients given above imply the following qualitative characteristics of the crack growth in the system in question. When the critical load is reached, the crack may be found to be in a stable or unstable state. In the first case the crack will increase instantaneously in length and pass into another stable state. Any further increase in its length will require additional load.

Since $K_{\mathrm{I}}^a = 0$ on the line $a = a_1(b)$, the crack cannot cross this line, i.e. its left tip is retarded. The right tip grows without bounds with increasing load (but in a stable manner).

3. Solution of the problem of a partly closed cut. If the condition $a < a_1(b)$ holds for the initial cut, then the cut will be partly closed. The closed part in which the relative displacement of the edges in the only feature, is adjacent to the left tip. We deal with this case by solving the problem of a partly closed cut. We shall assume that the cut edges are closed on $[a, c]$ and stress free on $[c, b]$. The geometry of the inclusion and the cut follows that of Sect. 1. The condition (1.2) holds on $[c, b]$, and on $[a, c]$ we adopt the condition of frictionless overlap

$$[p_{12}] = [p_{22}] = [u_2] = 0, \quad p_{12} = 0 \tag{3.1}$$

Since the continuation formulas (1.3) are governed only by conditions (1.1), at the inclusion boundary, they, remain unchanged.

Following [2], we introduce the function $\Omega = z\Phi' + \Psi$, and express the stresses and displacements in terms of this function.

From the first equations of (3.1) it follows that the function $Z = 2\Phi + \Omega$ remains continuous during the passage across $[a, c]$. The last equation of (3.1) yields the following expression for Ω on $[a, c]$:

$$\Omega - \overline{\Omega} + \varepsilon\,(G' + G^{\circ\prime} - \overline{G'} - \overline{G^{\circ\prime}}) = 0 \tag{3.2}$$

Expressing the condition (1.6) on $[c, b]$ in terms of Ω and separating the complex conjugate, we again arrive at (3.2). Maintaining the same accuracy as in Sect. 1, we obtain for Ω

$$\Omega(z) = \frac{1}{2\pi i X(z)} \int_a^b X(t)\,(-\varepsilon G^{\circ\prime} + \varepsilon \overline{G^{\circ}}{}') \frac{dt}{t - z} \tag{3.3}$$

The condition on $[c, b]$ yields for the function Z, in the same approximation, the Dirichlet problem independent of the point $z = a$

$$Z + \overline{Z} = -\varepsilon G^{\circ\prime} - \overline{\varepsilon G^{\circ\prime}} \tag{3.4}$$

and its solution is given by the formula

$$Z = \frac{1}{2\pi i \sqrt{(z - c)(z - b)}} \int_c^b \sqrt{(t - c)(t - b)}\,(-\varepsilon G^{\circ\prime} - \varepsilon \overline{G^{\circ\prime}}) \frac{dt}{t - z} \tag{3.5}$$

In order for the solution (3.3), (3.5) to have a physical meaning, we must have

$$p_{22} < 0, \quad \forall z \in [a, c) \tag{3.6}$$

$$p_{22}\ (v = c) = 0 \tag{3.7}$$

When $z \to c + 0$, (3.7) follows from (1.2). When $z \to c - 0$, the stress p_{22} has a singularity of the order of $(z - c)^{-1/2}$, with the concentration coefficient given by the real part of K_{I}^a from (1.7) in which a is replaced by c. It follows therefore that either $b = c$ or $c = a_1(b)$. In the first case the whole crack is closed and from (3.6) it follows that $|a| < l$ and $|b| < l$. This case shall be omitted.

When the point c lies on the line $a_1(b)$, the condition (3.6) can be verified directly. The local state of stress at the left tip is transverse shear. The corresponding stress concentration coefficient is determined in terms of the function Ω only, i.e. it does not depend on the point $z = c$. The quantity K_{II}^a can be found from the imaginary part of the first formula of (1.7). The root of (2.1) determined by the condition $|a| < h / \sqrt{3}$ gives the line $a_4(b)$ on which $K_{\mathrm{II}}^a = 0$. It follows that the crack cannot cross this line.

At the right tip $z = b$, the local state of stress in a combination of tension and transverse shear. The quantity K_{I}^b is given by the function Z, i.e. it does not depend on the closed segment, and the formula for $K_I{}^b$ is given by (1.7) in which a is replaced by c, while c and b are connected by the following relation:

$$b = \frac{c\,(c^2 - 3h^2)}{h^2 - 3c^2}$$

The quantity $K_{II}{}^b$ can be found from the imaginary part of the second formula of (1.7), and does not depend on the point $z = c$.

If the condition of fracture hold at the left tip $z = a$, then the crack will propagate along its initial direction. If on the other hand the condition of fracture holds at the right tip $z = b$, then the crack will deviate from its initial direction.

We can consider a symmetrical problem analogous to that discussed in Sect. 2. Then at the compressed tip we shall have pure shear, $K_{(2)}^a = 2K_{II}{}^a$. Pure tension will appear at the right free tip, and $K_{(2)}^b = 2K_I{}^b$. The quantity $K_I{}^b$ is independent of the point $z = a$, i.e. the magnitude of the critical load and continued growth of crack in the direction of the free tip will depend only on the initial value of b. The growth, which may be unstable during the initial stage, will undoubtedly become stable. The compressed tip may approach asymptotically the line $a_4\,(b)$ with the increasing load.

4. Discussion of results. Let us inspect the results obtained, in the light of the experimental data relating to the fracture of concrete. Let us quote briefly the necessary information concerning the fracture of concrete prisms under a central compressive force [11]. When the loads are small, the material shows isotropic and linearly elastic properties. At a certain value of the load (usually denoted by $R_T{}^\circ$) the microdefects which were present in the uncompressed sample, begin to grow. A large number of small cracks appears in the direction parallel to the applied load. Further increase in the value of the load is accompanied not so much by a growth of separate cracks, as by an increase in the number of the initial microcracks and by their stable growth.

The sample reduces in volume linearly up to the level of $R_T{}^\circ$. Beyond the level of $R_T{}^\circ$ the volume decreases at a slower rate and reaches its minimum value under the load $R_T{}^\vee$. Beyond $R_T{}^\vee$ the volume begins to increase (due to the expansion of cracks) and, when the applied load is increased further, the microcracks develop into large size cracks and this leads to a disintegration of the sample when the load reaches the value R_*. The numerical values of the parameters are $R_T{}^\circ \sim (0.2-0.4)R_*$, $R_T{}^\vee \sim (0.7-0.9)R_*$.

It is evident that when the system of defects in the load-free case is not extensive, then the behavior of a crack during the initial stages of loading is determined by its interaction with the nearest inclusions. For this reason the results obtained above should be compared with experiment at the load levels of the order of $R_T{}^\circ$. When the loads are of the order of $R_T{}^\vee$ and higher, the stability of the system of cracks is determined by their interaction and a different type of approach becomes necessary.

We see that the model discussed above has furnished us with two, basic, qualitative features; the change from the unstable to the stable growth of cracks and the appearance of a minimum in the critical load curve physically equivalent to the parameter $R_T{}^\circ$.

Using the concepts of similarity and the solutions obtained above, we arrive at the following formula for $R_T{}^\circ$, analogous to that given in [12] for composite materials:

$$R_T^\circ = K_c k f(\alpha) / \sqrt{R}$$

Here k is a coefficient which depends on the ratio of the elastic moduli of the filler and the matrix, α denotes the concentration of the coarse filler and R is the characteristic dimension of the filler.

Let us estimate the critical load for a typical crack. For the short length cracks $(b - a \ll R)$ we set $b - a = 10^{-1}$ mm, $K_c^2 = 10^{-1}$ kg^2/ mm^3,

$$\varepsilon = {}^1/_7, \ (a^2 + h^2)^2 / (R^2 (a^2 - h^2) \sqrt{2\pi}) = 1.$$

This yields $p = 7$ kg/ mm^2.

The value obtained exceeds the experimental value by two to three times. This can be explained as follows. First, only two nearest inclusions were taken into account and second, the parameter ε reflects only the difference between the Poisson's ratios while in a real material the shear moduli are also different and this leads to reduction in the value of the critical load. We should also note that the model in question can be used to provide an analytic explanation of the ultrasonic method of determining the point R_T° [11].

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